

The Fine Structure of Fibers and Crystalline Polymers.

III. Interpretation of the Mechanical Properties of Fibers

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In the past, interpretations of the mechanical properties of fibers in terms of molecular structure have usually been based on the theory of oriented fringed micelles. In this are envisaged the deformations taking place in the more irregular noncrystalline material, limited to some extent by the presence of blocks of rigid crystalline material. A typical schematic structural picture¹ is shown in Figure 1. Some authors^{2,3} have taken account of larger-scale features of the organization of vegetable fibers.

As has been pointed out, however, in the previous two papers of this series,^{4,5} the fringed micelle structure now appears to be unsatisfactory in some ways as a view of the fine structure of many types of fiber, and a fringed fibril structure has been suggested as an alternative. In this paper, the mechanical properties of fibers are reinterpreted in terms of fringed fibrils, and it is shown that a fibrillar structure gives a good explanation of the behavior of most fibers.

BEHAVIOR OF PLANT FIBERS WITH SPIRAL STRUCTURE

Mechanism of Deformation

It is convenient to deal first with the natural cellulosic fibers which have a spiral structure with mean helix angles ranging from about 6° in flax to about 30° in cotton and to more than 40° in coir and some selected leaf fibers. In the fringed fibril theory the structure may be regarded schematically as in Figure 2, the crystalline fibrils embedded in (and molecularly continuous with) a matrix of noncrystalline regions. The orientation of the molecules is along the fibrils. On extension of a fiber with this structure, deformation may occur in the following ways: (1) by an increase in length of the fibrils and of the noncrystalline regions in between, (2) (a) by extension like a spiral spring, with bending and twisting of the fibrils accompanied by (b) a reduction in volume of the fibrils, of the interfibrillar matrix of noncrystalline material, and of any void spaces present, and (3) by shearing of the noncrystalline regions to conform with the new configuration of the fibrillar structure.

The contributions of these various effects are summarized in Table I.

TABLE I
Contributions to Modulus in Deformation of Helical Fiber Structure

	In crystalline fibrils	In noncrystalline matrix
1. Due to increase in length	$\gamma E_c F_1(\theta)$	$+(1 - \gamma) E_n F_1(\theta)$ (small)
2. (a) Due to extension of spring	$\gamma \left(\frac{r}{R}\right)^2 \cos^2 \theta \frac{n_c E_n}{E_c \sin^2 \theta + 2n_n \cos^2 \theta}$ (negligible)	
(b) Due to volume change	$+\frac{\alpha^2}{\gamma} k_c (1 - 2 \cot^2 \theta)^2$ (small)	$+\frac{(1 - \alpha)^2 k_n}{1 - \gamma} (1 - 2 \cot^2 \theta)^2$
3. Adjustment between fibrils	—	$+X_n$ (probably small)

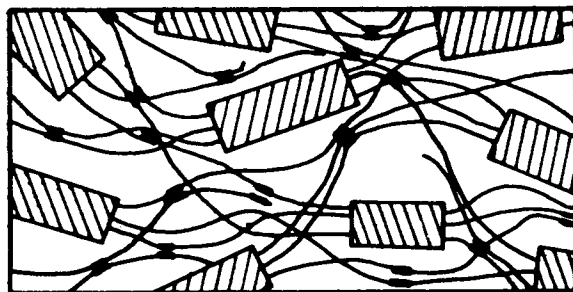


Fig. 1. Schematic view of fringed micelle structure used in interpreting mechanical properties of fibers.¹

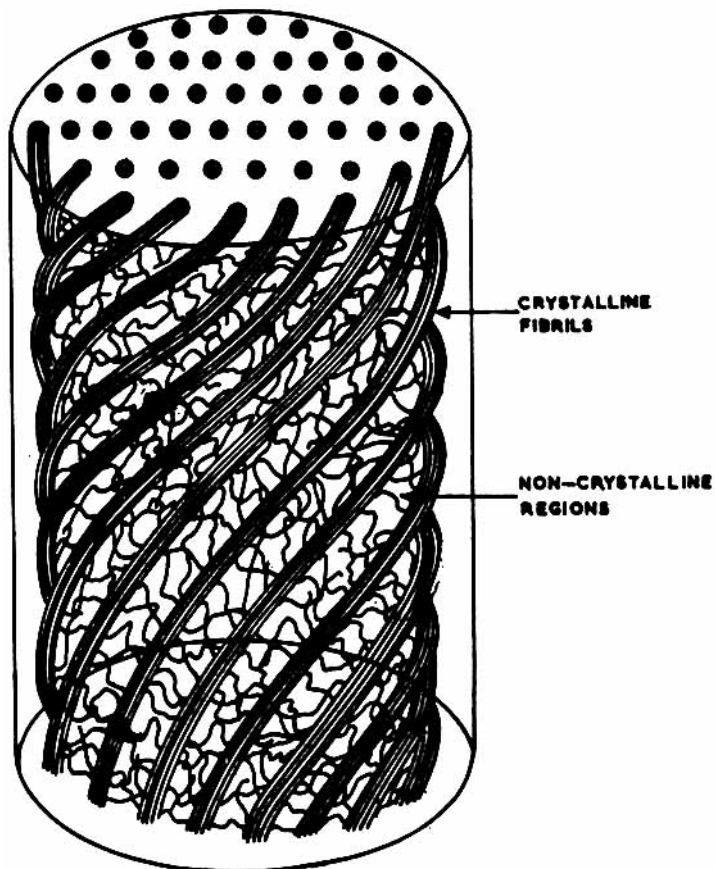


Fig. 2. Helical arrangement of fibrils in natural cellulose fiber.

Deformation by Stretching

The first type of deformation is similar to the extension of a twisted continuous filament yarn and would be expected to fit in with the theories which have been shown to apply to yarns.^{6,7} This means that the effec-

tive modulus of the twisted structure would be $EF_1(\theta)$, where E is the modulus of the untwisted structure and $F_1(\theta)$ is a function of the helix angle θ .^{*} In the simplest theoretical treatment,^{8,9} $F_1(\theta) = \cos^2\theta$, but Hearle^{7,10} has put forward more detailed theories having more complicated functions. In this paper $F_1(\theta)$ will be taken to equal the function which is referred to as $F_2(\alpha, \sigma_1, \sigma_2)$ in Hearle's second theoretical paper.⁷ This function shows the best fit with the experimental results for twisted yarns. It is assumed that the volume of the twisted structure remains constant during the deformation, as illustrated in Figure 3.

The combined effects of deformation due to stretching in both crystalline and noncrystalline regions can be covered by putting $E = \gamma E_c + (1 - \gamma)E_n$ where γ = volume of crystalline material per total fiber volume, E_c = Young's modulus of crystalline regions, parallel to the fibril axis, and E_n = Young's modulus of noncrystalline regions. It is possible that E_n will be much smaller than E_c so that the second term will be almost negligible and the effective modulus would be γE_c , that is, about $2/3 E_c$ in natural cellulose fibers.

Deformation in the Manner of a Spring

It is shown in Appendix I that, for the extension of each fibril like that of an independent spiral spring, the effective modulus would be:

$$\gamma(r/R)^2 \cos \theta [n_c E_c / (E_c \sin^2 \theta + 2n_c \cos^2 \theta)]$$

where r = fibril radius, R = helix radius, and n_c = shear modulus. The last term, in brackets [], must be less than either n_c or E_c , whichever is greater, and if, as is probable, $n_c \ll E_c$, then it tends to $n_c/\sin^2\theta$. The factors γ and $\cos \theta$ will have values slightly less than 1. However, the second term, $(r/R)^2$, will equal 10^{-6} for a fibril of radius 100 Å. at the outside of a fiber of radius 10 μ , and this means that the modulus will be very small except for an infinitesimal area at the center of the fiber; thus the resistance to deformation due to the stretching of fibrils like springs will be so small as to be completely negligible.

However, the springlike extension with a constant fibrillar length involves a reduction in volume. In an ordinary spring this can happen because the center of the spring is open, but in a closely packed array of springs, as in the spiral fibrillar structure, there will be a resistance to volume reduction. It is shown in Appendix II that if the volume changes were the determining factor we should have:

$$\text{Effective modulus} = K(1 - 2 \cot^2 \theta)^2$$

* In the rather general analysis of this paper, the problem of the exact definition of θ is not examined. In the theories on twisted yarn the angle used is α , the surface angle of twist in a structure in which the length of one turn of twist is constant throughout the cross section so that the helix angle decreases from α at the outside to zero at the center of the yarn. In natural plant fibers it may usually be more correct to take θ as constant throughout the cross section: this will cause $F_1(\theta)$ to decrease somewhat more rapidly with θ .

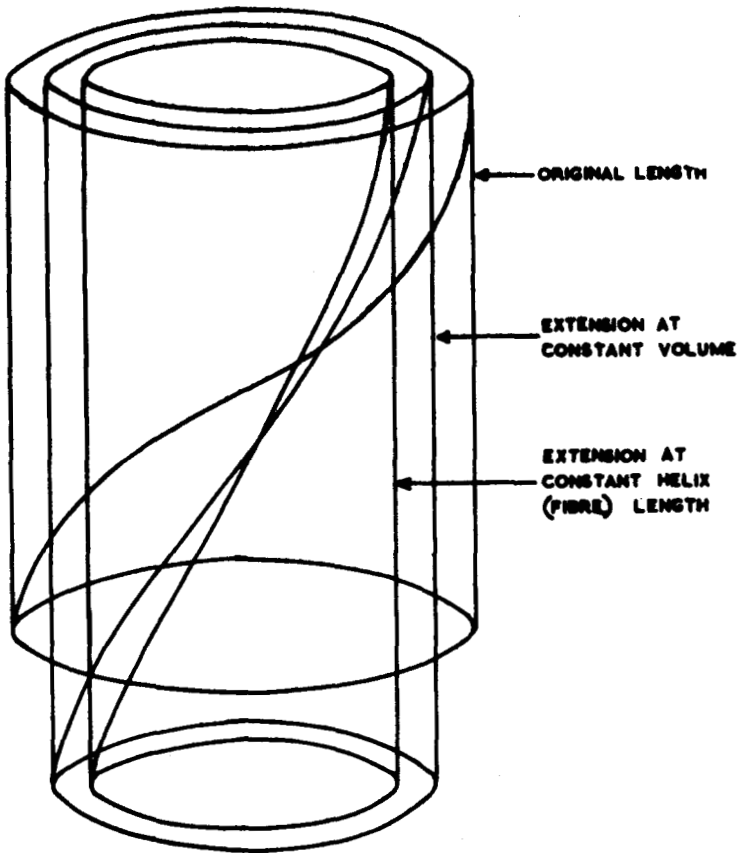


Fig. 3. Types of deformation in twisted structure.

where $K = [(1 - \alpha)^2 / (1 - \gamma)]k_n + (\alpha^2 / \gamma)k_c$, k_n being bulk modulus of noncrystalline regions, k_c bulk modulus of crystalline regions, and α the fraction of total volume change occurring in noncrystalline regions, or $\gamma k_n / [(1 - \gamma)k_c + \gamma k_n]$.

If, as would be expected, $k_c \gg k_n$, i.e., if the fibrils can be regarded as incompressible, then $\alpha \ll 1$ and $K = k_n / (1 - \gamma) \approx 3k_n$ for a natural cellulose fiber.

The above argument is based on the assumptions that (a) the fiber cannot untwist when it is extended and (b) the fiber is a solid cylinder. If either of these conditions is not satisfied, then the forces developed will be less. If untwisting is possible, as it may well be locally in cotton, where the direction of the spiral alternates frequently, then the volume reduction will be less and, in some circumstances, could be zero.

If there is a void space at the center of the fiber, the volume reduction could occur by a reduction of the volume of this void space, and the compression of the fiber material would be much decreased. However, such

a deformation is likely to be resisted by other smaller forces, and a factor proportional to $(1 - 2 \cot^2 \theta)^2$ will still represent the magnitude of the deformation which has to be accommodated. The effect of void spaces can therefore be regarded as causing a reduction in the effective value of K , the parameter including the bulk moduli.

Shear Deformation in Noncrystalline Regions

In considering the extension of a twisted yarn or of a system of spiral springs, it is tacitly assumed that the individual filaments are free to slide over one another in adjusting themselves to the new conformation. This will not be possible in the fringed fibril structure, because of the continuity of the molecular network through crystalline and noncrystalline regions. Some shear deformation of the noncrystalline matrix must take place, and this will add a further resistance to the extension of the fiber. The contribution will not be easy to calculate, but it will probably not be a large one. It has been included in Table I as the quantity X_n .

Combination of Mechanisms

Neglecting the last factor X_n , we see that there are two possible mechanisms of deformation of the spiral structure (1) due to an increase in length of fibrils at constant volume, with a modulus of $EF_1(\theta)$, and (2) due to springlike extension at constant fibril length, accompanied by volume com-

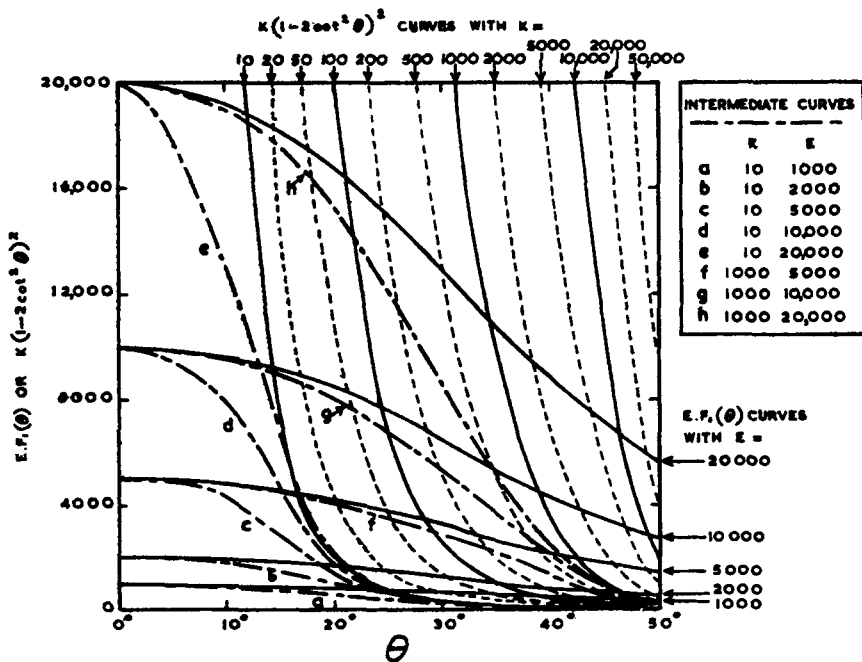


Fig. 4. Graphs of $EF_1(\theta)$ and $K(1 - 2 \cot^2 \theta)^2$ versus θ .

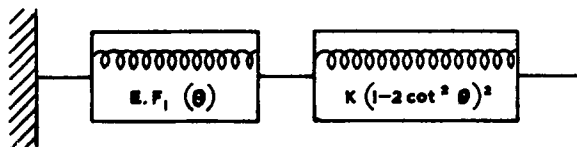


Fig. 5. Combination of two mechanisms in series.

pression, with a modulus $K(1 - 2 \cot^2 \theta)^2$. These two functions are plotted for various values of E and K in Figure 4. It will be seen that for low values of θ , $K(1 - 2 \cot^2 \theta)^2 \gg EF_1(\theta)$, showing that the springlike deformation with volume reduction can be ignored, and the fibril modulus will equal $EF_1(\theta)$. But $K(1 - 2 \cot^2 \theta)^2$ is much more sensitive to θ than is $EF_1(\theta)$, so that at high values of θ where $K(1 - 2 \cot^2 \theta)^2 \ll EF_1(\theta)$ the springlike mechanism will predominate and the fiber modulus will equal $K(1 - 2 \cot^2 \theta)^2$.

The two mechanisms will act like two springs in series, as in Figure 5, each tending to relieve the stresses in the other by an interchange of strain, and so for intermediate values of θ we shall have: Effective modulus due to combined factors

$$= \frac{EF_1(\theta) [K(1 - 2 \cot^2 \theta)^2]}{EF_1(\theta) + K(1 - 2 \cot^2 \theta)^2}$$

This form of equation will not necessarily be exactly correct, since the interchange of strain between the two mechanisms is not quite as simple as between two springs in series because of the geometry of the system, but it should be a reasonable approximation for intermediate values. A selection of intermediate curves is included in Figure 4.

Comparison with Experiment

In Figure 6, eq. (4) has been fitted to values found by Spark et al.¹¹ for the modulus of two sisal fiber cells having spiral angles of 10 and 42° respectively. For the fitted curve, $E = 2180$ and $K = 229$ Kg./mm.². Neglecting the contribution of the noncrystalline regions, we should have $E = \gamma E_0 \approx 2/3 E_0$. Treloar¹² has calculated a value of 5770 kg./mm.² for the modulus of cellulose crystals, giving $2/3 E_0 = 3900$ kg./mm.². The fitted value of E is of the right order of magnitude, although somewhat lower than the theoretical value (which was itself considered by Treloar to be too low an estimate). The difference may, in part, be explained by the occurrence of creep: dynamic values of modulus of sisal are appreciably higher.

If the fibrils are assumed to be incompressible, K should equal $k_n/(1 - \gamma) \approx 3k_n$. This fitted value of K would thus correspond to a bulk modulus in the noncrystalline regions of 76 kg./mm.². It is rather low in comparison, for example, with a bulk modulus of about 200 kg./mm.² for a soft rubber, owing partly, perhaps, to the fact that the full volume

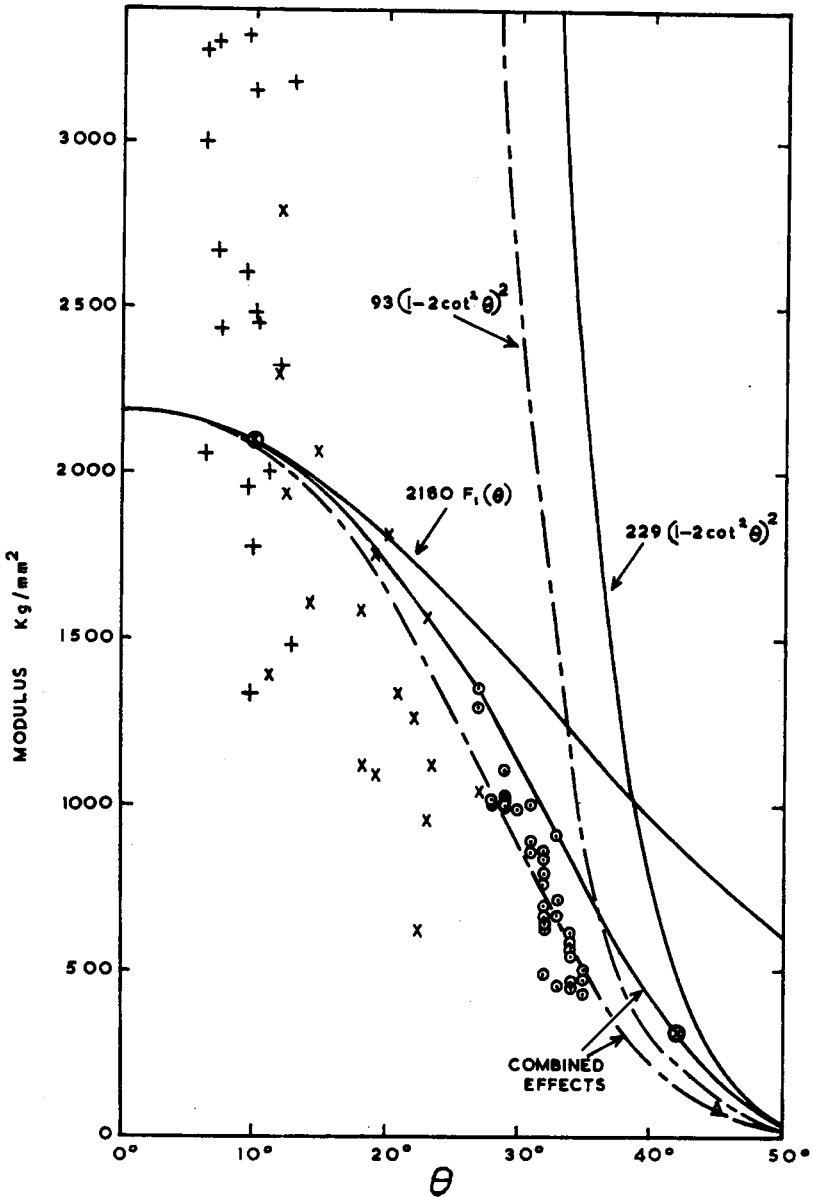


Fig. 6. Comparison of theory and experiment for various fibers: leaf; (⊗) sisal; (⊙) cotton; (×) leaf; (+) bast; (Δ) coir.

change in the material does not occur because of untwisting or a reduction in void space at the center of the fiber. Barkas¹³ has pointed out that the saturation moisture absorption of wood, which is reduced from about 22%, when swelling is allowed, to 13% at constant gross volume, is limited

to about 1.5% when the volume of the wood cell wall is held constant; this illustrates the great potentialities for expansion into the void space.

It may also be noted that increasing the value of k_n to a more reasonable value, i.e., 200 kg./mm.², so that $K = 600$, would give a modulus of 520 kg./mm.² at 42° as against the experimental value of 315 kg./mm.². Considering the crudity of the analysis this is still a reasonable fit.

Included in Figure 6 are values of the modulus of a variety of cotton fibers measured by Meredith² and of some bast, leaf, and coir fibers measured by Stout and Jenkins.³ They are not very far from the theoretical curve

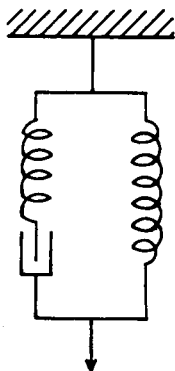


Fig. 7. Eyring's three-element model of two ideal springs and nonlinear dashpot.

combining the two mechanisms, except that there is a very wide spread of values for the bast fibers. Some of these results will be influenced by the fact that the commercial bast and leaf fibers are made up of an aggregate of plant cells and contain varying amounts of lignin, pectin, and other materials. The values for cotton are rather better fitted by the curve with $K = 93$.

Influence of Time and Moisture

The mechanical behavior of fibers is time-dependent. Provided the stresses are not too great, the best agreement between experiment and theory has been the three-element model given by Tobolsky and Eyring,¹⁴ Figure 7, which incorporates, on the basis of reaction rate theory, a dashpot with a hyperbolic sine law of flow. Such a model could be interpreted according to the ideas given above in the following way. The arm containing the spring would represent the elastic extension of the crystalline fibrils, given by the term $\gamma E_c F_1(\theta)$ in Table I, and to a lesser extent by the compression of fibrils, that is, the term $(\alpha^2/\gamma)k_c(1 - 2 \cot^2\theta)^2$. The other arm would represent the viscoelastic compression of the noncrystalline material, given by the term $[(1 - \alpha)^2/(1 - \gamma)]k_n(1 - 2 \cot^2\theta)^2$ and also by the contributions of other deformations in the noncrystalline regions, that is, the terms $(1 - \gamma)E_n F_1(\theta)$ and X_n . There may also be some time-

dependent element in the deformation of the crystalline fibrils due to rearrangement of the molecules in the imperfect parts of the crystal.

When the stresses become larger, secondary (nonrecoverable) creep also occurs, so that Eyring's model ceases to be valid. This secondary creep would be a further feature of the deformation of the noncrystalline regions.

The moduli of plant fibers decrease as water is absorbed. Since the water does not penetrate the crystalline regions, this can be due only to changes in the interfibrillar material. In cotton and other fibers with high spiral angle, it can be explained by a change in k_n , the bulk modulus of the noncrystalline regions. However, the change is as marked in a highly oriented fiber, such as ramie, as it is in cotton,¹⁶ showing that the noncrystalline regions are playing an important part even where the extension of the fibrils appears to be the dominant mechanism. This must be due partly to the influence of the second term $(1 - \gamma)E_n$ in the expression for E —in other words, in the dry fibers there is appreciable resistance to extension of the noncrystalline regions, with E_n comparable to E_c —and partly to the influence of the neglected factor X_n , that is, the shearing forces arising from the fitting of the noncrystalline regions into the new configuration, which will also be expected to be much greater in a heavily crosslinked, dry structure. Another factor which will have some effect is the decrease in spiral angle resulting from the swelling of the fiber.

Fiber Breakage

One may expect break to occur when the stress concentrations rise above a certain level, so that, in general, the fiber strength should decrease with spiral angle in the same way that the modulus decreases. This is, in fact, found, although there is a good deal of scatter which reflects the varying incidence of flaws and points of weakness in the structures of different fibers.

If the fibers are dry, the comparative rigidity of the noncrystalline material will restrict the freedom of movement of the fibrils and so promote high stress concentrations whereas, when the fiber is wet, there will be greater freedom and lower stress concentrations and the fiber strength will increase, as is found experimentally in the natural cellulosic fibers. A similar effect was found in model yarns when the strength fell on glueing together of the filaments.¹⁶

APPLICATION TO OTHER FIBERS

The Effects of Irregular Orientation

The spiral arrangements of fibrils in plant fibers is a convenient model to study because of the regularity of the geometry. In a man-made fiber, one can expect an irregular arrangement of the fibrils oriented more or less parallel to the fiber axis, as in Figure 8a. However, deformation can still

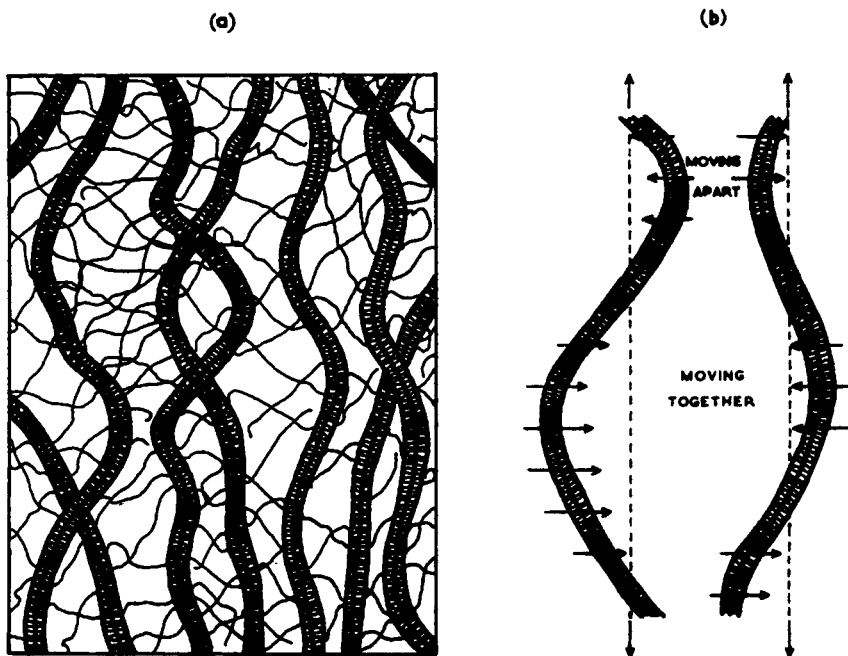


Fig. 8. (a) Irregular orientation of fibrils. (b) Movements of fibrils apart and together in different sections, as the fibrils are straightened.

take place in the two ways discussed for the spiral structure, namely by a simple extension of the fibrils and interfibrillar material (without change of volume) and by a straightening of the fibrils without a change in their lengths. The first mechanism will be resisted directly by the force needed to extend the fibrils; the second mechanism will be resisted chiefly by deformation in the noncrystalline regions. The nature of the latter deformation will be complicated, but it is clear that, whether or not there is an overall volume reduction, there will certainly be local distortions as some neighboring fibril elements come closer together while others move farther apart (see Fig. 8b).

It seems reasonable, therefore, to apply similar theoretical ideas to these fibers, the spiral angle θ being replaced by an angle which is related to the mean angle of orientation of the fibrils (the crystalline regions), and the two functions modified to take account of the difference in structure and consequently in deformation. It is, of course, well known that the modulus of man-made fibers decreases as the degree of orientation decreases, but there is a lack of data adequate to test the theory.

Regenerated Cellulose Fibers

The interpretation of the results for rayon fibers is rather complicated. In ordinary rayon fibers the modulus falls to an extremely low value when the fiber is wet, although there is little penetration of water into the crystal-

line regions. Extending dry rayon is similarly very easy once the yield point has passed and the crosslinks in the noncrystalline regions are breaking. There appear two possible explanations of this behavior: (a) the deformation may be due to a straightening of the fibrils resisted mainly by shearing forces in the noncrystalline regions, which would be very low in a wet fibre, or (b) the structure may not be fibrillar but may resemble the fringed micelle structure, so that the influence of the noncrystalline regions would be dominant. There is some other evidence⁴ that ordinary rayon fibers are not fibrillar in structure.

In contrast to ordinary rayon fibers, Fortisan and the new "polynosic" rayons have a high wet modulus; the decrease in modulus on wetting is somewhat greater than in cotton, but this may be explained by the greater proportion of noncrystalline material present. The modulus of dry polynosic fibers is also much higher than that of ordinary rayon fibers, and the yield point is not as marked. These fibers are made by processes in which regeneration and crystallization follow the solidification and stretching of the fibers, and they almost certainly have a fibrillar structure. Their behavior would fit in well with the fibrillar explanation of mechanical properties developed in this paper, and their difference from ordinary rayons is additional evidence that the latter do not have a fibrillar structure.

Synthetic Fibers

In synthetic fibers another factor is involved. After drawing, these fibers (except perhaps for the acrylic fibers) may have a more or less oriented fibrillar structure, and so the theory described in this paper would be expected to apply. However, on releasing the drawing tension the oriented noncrystalline regions will tend to contract as the molecules take up more irregular configurations: this is an expression of rubberlike elasticity and will become particularly marked if the fiber is heated free of tension. The contraction of the noncrystalline regions will be opposed by the crystalline fibrils and a balance will be achieved. This balance of opposing forces means that the modulus of the fibers will be low. The effect should be very important in nylon, polyethylene, and polyvinyl chloride fibers, where the chain molecule is so flexible that it is easily kinked into a tightly coiled, random configuration, but it should be less marked in the case of a stiff molecule such as that of polyethylene terephthalate, which cannot easily bend. The modulus of Terylene (polyethylene terephthalate) is, in fact, much greater than that of the other fibers mentioned, which all show a considerable reduction in modulus on being heated in water at 95°C.¹⁷

Protein Fibers

In wool and similar protein fibers the mechanical properties are largely determined by the highly specific molecular mechanism involved in the transformation from the α helix to the extended β chain. In silk the stress-strain curve is rather similar to that of Terylene, and the fibrillar theory should apply.

CONCLUSION

The present paper has shown that the mechanical properties of most fibers can be reasonably interpreted in terms of a theory which views their structure as an assembly of fringed fibrils. The deformation of plant fibers having spiral structure can be explained as a combination of (1) stretching of the fibrillar structure without volume change, and (2) stretching of the spiral arrangement, as a spring, without changing fibrillar length but with a reduction in volume, which resists the springlike extension. Similar ideas should be applicable to the deformation of irregularly oriented, man-made fibers, although the mathematical analysis may be more complex and remains to be worked out.

The agreement between experiment and theory for the plant fibers is quite good, but it is worth while to summarize here the errors and approximations involved in the theoretical treatment:

1. The spiral geometry has been considered only in general terms, and the effects of such details as the variation in spiral angle through the fiber have not been worked out.

2. Some contributions to the resistance to deformation have been neglected. This is probably quite justified for the resistance to springlike extension of the fibrils themselves, but may not be so for the other effects in the noncrystalline regions represented by the term X_n in Table I.

3. There is some lack of rigor in the theoretical proofs, particularly in the equation combining the two effects.

4. If untwisting of the fibers, or portions of the fibers, occurs, then the volume reduction will be less than calculated, but other resisting forces will be brought into play.

5. If there are void spaces, the volume reductions will be less but other resisting forces may come into play.

6. In fibers composed of an aggregate of plant cells the overall structure will play some part in determining the mechanical behavior.

Appendix I. Stretching of Fibrillar Structure Like a Spiral Spring

It is assumed that each fibril can extend like an independent spiral spring. Under a load W , the fractional extension is given by:¹⁸

$$\text{Fractional extension} = (2WR^2/\pi r^4)[(\sin^2\theta/n_c) + (2 \cos^2\theta/E_c)]$$

where R = radius of helix, r = radius of fibril, θ = helix angle, n_c = shear modulus, and E_c = Young's modulus.

The effective area of cross section of the fiber occupied by a single fibril is equal to $(1/\gamma)\pi r^2 \sec \theta$: the factor $1/\gamma$ is introduced to allow for an appropriate amount of noncrystalline material surrounding the fibril; the factor $\sec \theta$, to allow for the obliquity. Therefore, stress = $W/(1/\gamma)\pi r^2 \sec \theta$, giving:

$$\begin{aligned} \text{Modulus} &= [(\gamma W/\pi r^2/\sec \theta)(\pi r^4/2WR^2)]/[\sin^2\theta/n_c + 2 \cos^2\theta/E_c] \\ &= \gamma(r/R)^2 \cos \theta n_c E_c / (E_c \sin^2\theta + 2n_c \cos^2\theta) \end{aligned}$$

Appendix II. Forces Due to Volume Compression in a Model Composed of Inextensible Flexible Fibrils Embedded in Deformable Matrix

Consider a circular cylinder, as shown in Figure 9a, of length h and radius r with a fibril of length l spiralling around it and making an angle θ with a line parallel to the axis of the cylinder. An "opened-out" diagram of the surface of the cylinder is shown in Figure 9b. It is assumed that an extension Δh of the cylinder takes place by the fibrils' bending and twisting, like a spiral spring, the resistance to this deformation coming not from the

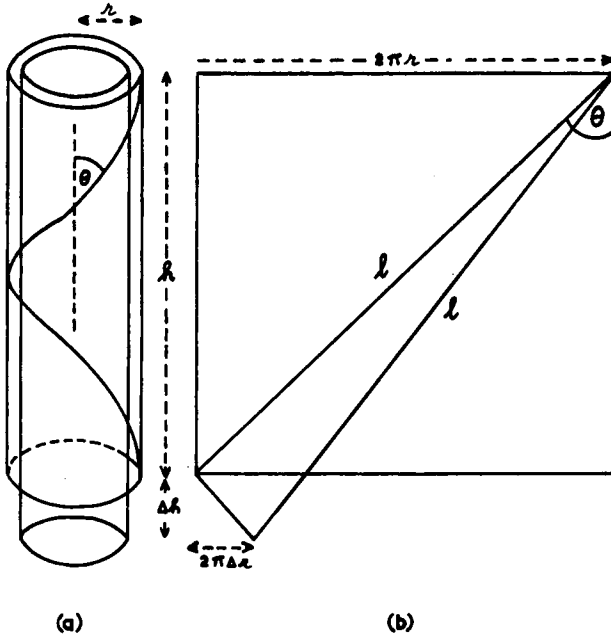


Fig. 9. (a) Spiral structure in a cylinder. (b) Planar view of cylinder surface.

force required to deform the fibrils but from the forces required to compress the material. It is also assumed that there is no increase in length of the fibrils.

In Figure 9b we see that:

$$4\pi^2 r^2 = l^2 - h^2$$

and, therefore, for $l = \text{constant}$ the change in radius Δr is given by:

$$8\pi^2 r \Delta r = -2h \Delta h$$

and neglecting terms in $(\Delta r)^2$ and $(\Delta h)^2$:

$$\Delta r = -(1/2\pi)(h/2\pi r)\Delta h = -(1/2\pi)\Delta h \cot \theta$$

The volume of the cylinder, V , is $\pi r^2 h$, and the change in volume ΔV , is given by:

$$\Delta V = \pi r^2 \Delta h + 2\pi r h \Delta r$$

and neglecting higher terms in Δr and Δh ,

$$\begin{aligned} \Delta V &= \pi r^2 h [\Delta h/h - (2h/r)(1/2\pi)(\Delta h/h) \cot \theta] \\ &= \pi r^2 h (\Delta h/h) (1 - 2 \cot^2 \theta) \end{aligned}$$

If the fibrils are relatively incompressible, most of this volume change will take place in the matrix. However, a correction for any volume change in the fibrils can be applied as follows. Let the volume change in the fibrils, with a total volume γV , be αV , leaving a change $(1 - \alpha)\Delta V$ in the interfibrillar volume $(1 - \gamma)V$. Consequently we have:

$$\begin{aligned} \text{Volume strain in matrix} &= (1 - \alpha)/(1 - \gamma)(\Delta V/V) \\ &= (1 - \alpha)/(1 - \gamma)(\Delta h/h)(1 - 2 \cot^2 \theta) \\ \text{Pressure in matrix} &= p_r = (1 - \alpha)/(1 - \gamma)k_n(\Delta h/h) (1 - 2 \cot^2 \theta) \end{aligned}$$

where k_n = bulk modulus of matrix.

Similarly, for the fibrillar material, we have:

$$\begin{aligned} \text{Volume strain in crystalline material} &= (\alpha/\gamma)(\Delta V/V) = (\alpha/\gamma) \\ &(\Delta h/h)(1 - 2 \cot^2 \theta) \end{aligned}$$

$$\text{Pressure in crystalline material} = p_c = (\alpha/\gamma)k_c(\Delta h/h)(1 - 2 \cot^2 \theta).$$

The internal pressure p_n and p_c must be balanced, so that we have:

$$\begin{aligned} (\alpha/\gamma)k_c(\Delta h/h)(1 - 2 \cot^2 \theta) &= (1 - \alpha/1 - \gamma)k_n(\Delta h/h)(1 - 2 \cot^2 \theta) \\ \alpha k_c/\gamma &= (1 - \alpha/1 - \gamma)k_n = k_c/[(1 - \gamma_c)k + \gamma k_n] \end{aligned}$$

Suppose a force F is required to stretch the cylinder by Δh ; then the work done ($= 1/2 F \Delta h$, assuming Hooke's law) must equal energy stored by compression of material. This gives:

$$\text{Work done} = 1/2 F \Delta h = 1/2 \Sigma p dV$$

where $\Sigma p dV$ is the product of pressure and volume change for the two types of material. But:

$$\begin{aligned} F \Delta h = \Sigma p \Delta V &= [(1 - \alpha)^2/(1 - \gamma)]k_n(\Delta h/h)^2(1 - 2 \cot^2 \theta)^2 (1 - \gamma)V \\ &+ (\alpha/\gamma)^2 k_c(\Delta h/h)^2(1 - 2 \cot^2 \theta)^2 \gamma V \\ &= \{ [(1 - \alpha)^2/(1 - \gamma)]k_n + (\alpha^2/\gamma)k_c \} [(\Delta h/h)^2 \\ &(1 - 2 \cot^2 \theta)^2 V] \end{aligned}$$

and:

$$\begin{aligned} \text{Modulus} &= E_2(F/\pi r^2)/(\Delta h/h) = (F \Delta h/V)/(\Delta h/h)^2 \\ &= \{ [(1 - \alpha)^2/(1 - \gamma)]k_n + (\alpha^2/\gamma)k_c \} (1 - 2 \cot^2 \theta)^2 \\ &= K(1 - 2 \cot^2 \theta)^2 \end{aligned}$$

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Synopsis

The mechanics of extension of plant fibers is considered in terms of a spiral arrangement of crystalline fibrils embedded in a noncrystalline matrix. Deformation may take place either by stretching of the fibrils or by extension such as that of a spiral spring. In the latter, the major resistance comes from the reduction in volume. The extension mechanism predominates for low spiral angles; the spring mechanism, for high spiral angles. There is reasonable agreement between the theoretical expressions and experimental results. The application of similar ideas to other types of fiber structure is considered.

Résumé

On considère le mécanisme d'extension de fibres naturelles comme un arrangement en spirale de fibrilles cristallines incrustées dans une matrice noncristalline. Il peut y avoir déformation soit par étirement des fibrilles, ou par une extension comme un ressort en spirale. Dans la dernière méthode, la plus grande résistance provient de la réduction de volume. Le mécanisme d'extension prédomine avec de faibles angles de spirale et le mécanisme du ressort pour des grands angles. Il existe un accord raisonnable entre les expressions théoriques et les résultats expérimentaux. On envisage l'application d'idées semblables à d'autres types de structures fibreuses.

Zusammenfassung

Der Mechanik der Dehnung von pflanzlichen Fasern wird eine Spiralanordnung kristalliner, in eine nicht-kristalline Matrix eingebetteter Fibrillen zugrunde gelegt. Deformation kann entweder durch Streckung der Fibrillen oder durch eine spiral-

federartige Dehnung zustande kommen. Bei letzterer stammt der Widerstand hauptsächlich aus der Volumsreduktion. Bei niedrigem Spiralwinkel überwiegt der Dehnungsmechanismus, bei hohem Spiralwinkel der Federmechanismus. Es besteht eine annehmbare Übereinstimmung zwischen den theoretischen Ausdrücken und den Versuchsergebnissen. Die Anwendung ähnlicher Vorstellungen auf andere Faserstrukturtypen wird in Betracht gezogen.

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